Experimentally tracking unstable steady states by large periodic modulation

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Experimental suppression of chaos has been achieved in an optically pumped far-infrared $^{15}NH_3$ laser which displays Lorenz-like chaos. The method of control involves the application of a large amplitude slow (i.e., nonresonant) modulation of the pump power. This may be related to a delayed bifurcation to chaos observed when the pump power is ramped at a constant rate. [S1063-651X(98)10301-X]

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I. INTRODUCTION

There has been a lot of effort in the last few years devoted to the possibility of changing the deterministic chaos in nonlinear dynamic systems into regular behavior. Such control of chaos falls into two categories; techniques where the dynamics of the system are perturbed using feedback [1,2], and techniques whereby no feedback is employed [3,4]. The feedback mechanisms can be subdivided into the categories of continuous [2] or discrete feedback [1], and generally involve the perturbative stabilization of an unstable point or orbit in phase space. The nonfeedback mechanisms can also be divided into two categories. An oscillation resonant or near resonant with a system instability may stabilize an unstable point or orbit in some perturbed part of the phase space [5,4]. The system dynamics may also be made stable in regions where chaos normally occurs in the unperturbed system by applying a slow, nonresonant change to it. Something similar was initially demonstrated by Mandel and Erneux [6], who applied a slow linear increase in time of the Rayleigh number in the Lorenz equations. They showed that the onset of the first bifurcation in the equations comes at larger Rayleigh number than for the unperturbed equations.

More recently Vilaseca *et al.* [7] have shown that a large-amplitude slow periodic modulation of a control parameter, namely, the pump power or the detuning, can in the complex Lorenz-Haken equations suppress chaos. They have shown, in the case of pump power modulation, that the output intensity of the laser almost exactly follows the intensity of the stationary solution corresponding to the instantaneous value of the pump power, as the pump power is modulated about the chaos threshold. The system remains close to (tracks) the steady state solution at any time instead of becoming chaotic when the pump crosses the chaos threshold. They also showed that the system is globally stable by the fact that the stability was unaffected by the value of the initial conditions. Their paper goes on to give a detailed account of the effect of modulating the detuning in the laser, achieving similar results. Although at the present time it is not possible with our experimental apparatus to modulate the detuning, we are able to modulate the pump power. Our experiments show that the suppression of chaos by modulating the pump power as presented by Vilaseca *et al.* [7] is indeed possible in the optically pumped far-infrared $^{15}NH_3$ laser when it displays Lorenz-like chaos. There is experimental evidence that the method of Vilaseca *et al.* can control dynamical regimes and inhibit chaos in a nonautonomous (i.e., driven) system [13].

II. EXPERIMENT

The experimental setup is shown schematically in Fig. 1. The most significant parts of the setup include the $^{13}CO_2$ laser pump and the method whereby its output is modulated. The rest of the setup is the same as has been previously reported by Win *et al.* [8]. The $^{13}CO_2$ laser pump power is controlled by the use of an acousto-optic modulator (AOM). The power of the rf traveling wave injected into the crystal determines how much of the incident light is diffracted. Amplitude modulating the rf drive therefore gives a simple means of producing a time varying pump power. We have used an arbitrary function generator to produce the complex functions shown in the experimental figures. In this way we were able to establish pump levels where the system was stable or unstable, and thereby bracket the chaos threshold level. Because slow drifts of pressures and cavity lengths can lead to long term uncertainties in the values of the system parameters we have tried as much as possible to set up scenarios where the control is demonstrated unambiguously within single short experimental records.

III. RESULTS

In order to demonstrate control of chaos the signal shown in the upper portion of Fig. 2 was applied to the pump. The signal is the amplitude of the diffracted light from the AOM, inverted to reflect the behavior of the pump. The signal shows an oscillation of frequency approximately 600 Hz followed by two periods of constant pump intensity. The modulation period is around 10^{-4} of the average frequency of the chaotic pulsations and compares well with the theory. The response of the laser to the two constant pump power levels shows that the chaos threshold is somewhere between these levels. The peak to peak amplitude of the oscillations shown in the figure is about 15% of the chaos threshold pump power. Control at such a low ratio is achievable with the complex Lorenz-Haken equations but is at the lower end of the range. However, whenever measurements of the chaos threshold of the laser are made it has always been higher than that shown by the theory. There is good reason to believe that one of the explanations for this is imperfect pump mode coupling to the laser mode in the cavity. It has clearly been shown that if the pump mode is not matched to the laser mode the pump power required to reach the chaos threshold increases markedly [9]. As the pump mode could never be perfectly coupled to the laser mode due to their differences in

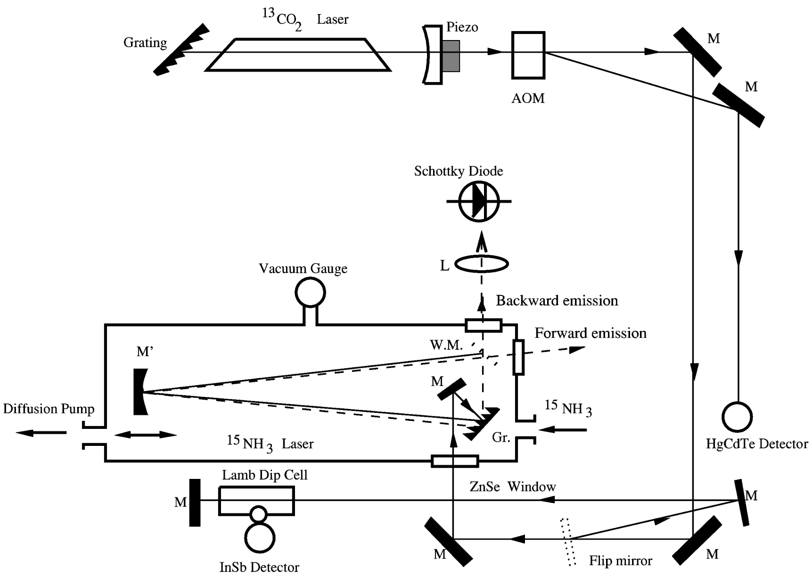

FIG. 1. The experimental setup showing the carbon dioxide pump laser, the ammonia laser, and the detectors used to monitor the pump power (HgCdTe detector) and frequency (Lamb dip cell) and the ammonia laser output intensity (Schottky diode).

wavelength an increase in the pump power required to reach chaos threshold follows and this results in a decrease in the ratio of modulation amplitude to threshold level.

In Fig. 2 the output of the laser clearly shows that the chaos threshold is somewhere between the two constant pump power levels and that the modulations oscillate about the chaos threshold. The oscillations show no chaos, indicating that the chaos has been suppressed in the manner shown by Vilaseca *et al.* Figure 3 is an expansion of a part of the chaotic region in Fig. 2 to show that the laser is producing Lorenz-like chaos with long spirals. This behavior has been observed in the laser for only a small region of tuning which is close to zero detuning. The pressure was found not to be critical with control being achieved as long as the pressure is above about 25 μbar. This is the region where the chaos in the laser approaches closest to true Lorenz chaos, with the double cusp observed in the return maps almost disappearing [9].

Whenever the behavior deviates from that shown in Fig. 2 the behavior shown in Figs. 4 or 6 appears. In the laser the dynamics always destabilize first on the negative slope of the modulation of the pump power and then the instability spreads to the positive slope as any parameter, such as detuning or pump power modulation intensity, are changed, away from stability. The region which becomes chaotic last is where the pump power modulation slope is maximum. Figure 5 shows a closeup of the region represented by the negative slope of the pump power in Fig. 4. The periodic behavior shown here is always precursory to the chaotic behavior displayed in Fig. 6. Even in Fig. 7, a closeup of the modulations in Fig. 6, the periodic behavior is present before the oscillations die away.

Vilaseca *et al.* [7] reported similar less than perfect tracking behavior where the laser detuning was modulated. We have examined numerical solutions of the complex Lorenz-Haken equations for zero detuning and found similar behav-

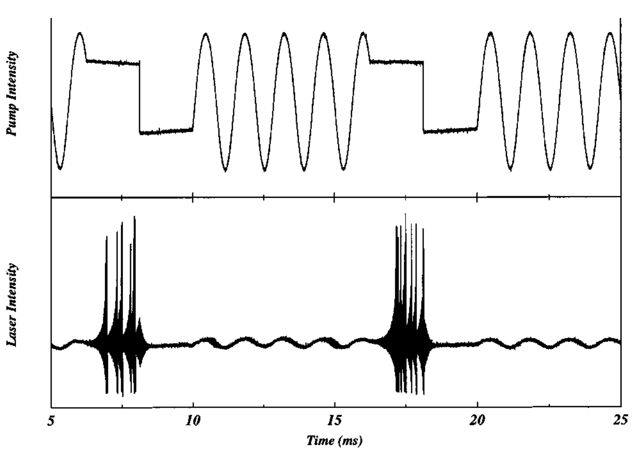

FIG. 2. Time evolution of the output intensity of the laser, the lower time trace, plus the corresponding pump power intensity in the upper time trace. Behavior during the two constant pump power periods indicates clearly that the chaos threshold must be somewhere between the two levels. The laser was operated at a pressure of 30 μbar. Amplitude units are in volts, which is proportional to intensity.

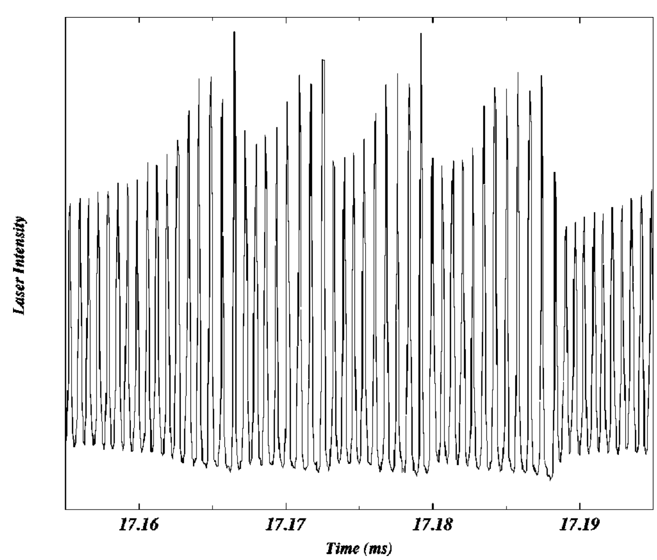

FIG. 3. Time evolution of a small portion of the intensity data in Fig. 2. The characteristic spiraling observed in Lorenz chaos is clearly present. Amplitude units are in volts, which is proportional to intensity.

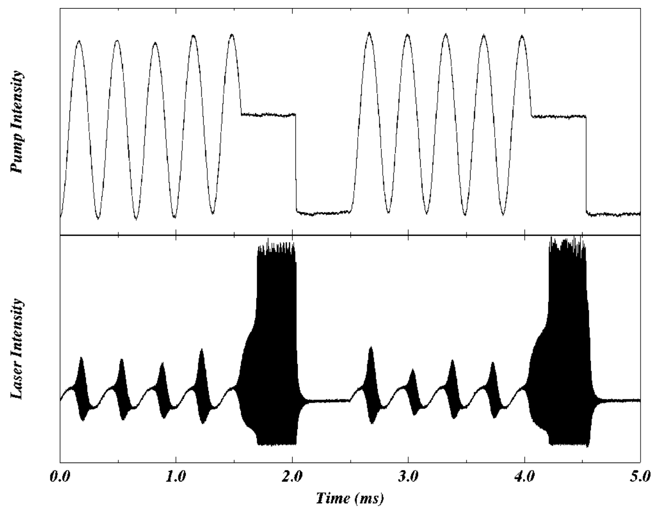

FIG. 4. The time evolution of the laser intensity and pump power intensity as shown in Fig. 2. The negative slope of the pump power modulation is periodic. The laser operated with all the same parameters as in Fig. 2 but with the laser slightly tuned away. Amplitude units are in volts, which is proportional to intensity.

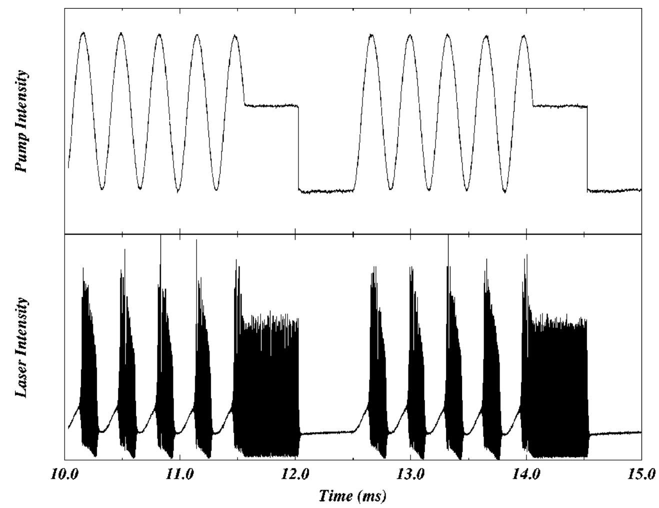

FIG. 6. The time evolution of the laser intensity and pump power intensity as shown in Fig. 2. The negative slope of the pump power modulation is chaotic and resembles Lorenz-like chaos in its spiral-like nature. The laser operated with all the same parameters as in Fig. 2 but with the laser tuned slightly more than in Fig. 4. Amplitude units are in volts, which is proportional to intensity.

ior as the pump power is increased or modulation amplitude is decreased. Figures 8(a)–8(d) show the transition to chaos as the modulation amplitude is changed from 2.3 to 1.5, for an average pump power of 14 which is at the chaos threshold. We hypothesize that the behavior shown in Fig. 8 is due to the delayed onset of the bifurcation to chaos which naturally occurs in the Lorenz equations [10,11]. When the Lorenz equations show metastable chaos, the phase space is divided into three basins of attraction, two of which contain stable fixed points and one of which has a chaotic attractor whose trajectories at one time intersect either basin of the fixed points to finish at one of them. Figure 8(a) shows a small period of preturbulence after which the trajectory spirals into the tracking fixed point, indicating that the limit cycle which denotes the boundary between the basins of attraction in the Lorenz equations exists well above the chaos threshold. Figure 8(b) shows the trajectory intersecting a stable basin on the positive slope trying to settle on one of the stable fixed points but not having settled down enough to stay within this basin on the negative slope. The trajectory henceforth spirals outward again into preturbulence. Once the trajectory is close enough to the stable fixed point as the slope of the pump power changes from positive to negative, the trajectory stays within the basin of the stable point and thereafter remains there. The unstable limit cycle which defines the border between the stable and unstable basins therefore must be smaller in size on the negative slope of the pump power modulation than on the positive slope. In fact in Fig. 8(c) the limit cycle no longer exists on the negative slope but obviously does on the positive slope. Figure 8(d) shows the attractor when chaos has fully developed. The main differences between the route to chaos in the laser and that shown in the equations is the appearance of the noncha-

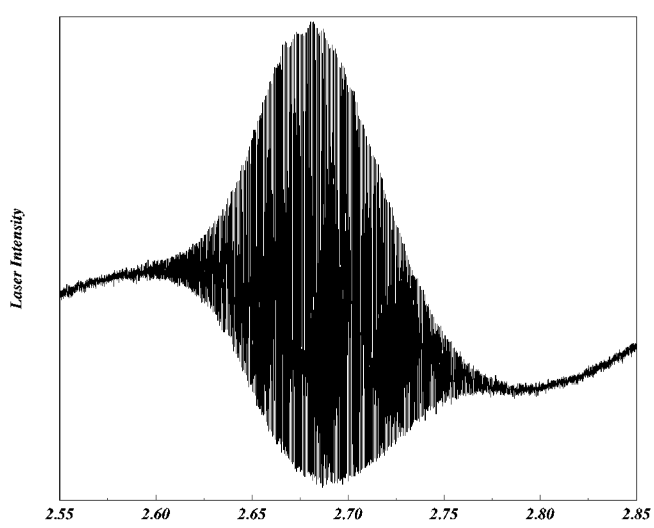

FIG. 5. A closeup of the output intensity shown in Fig. 4. Amplitude units are in volts, which is proportional to intensity.

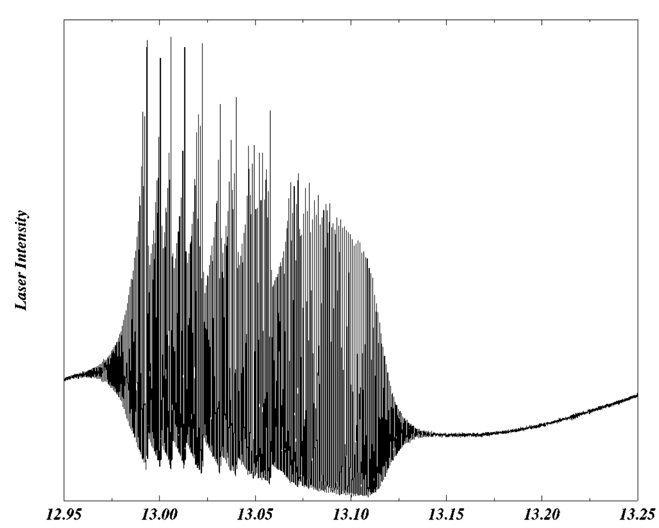

FIG. 7. A closeup of the output intensity shown in Fig. 6. Amplitude units are in volts, which is proportional to intensity.

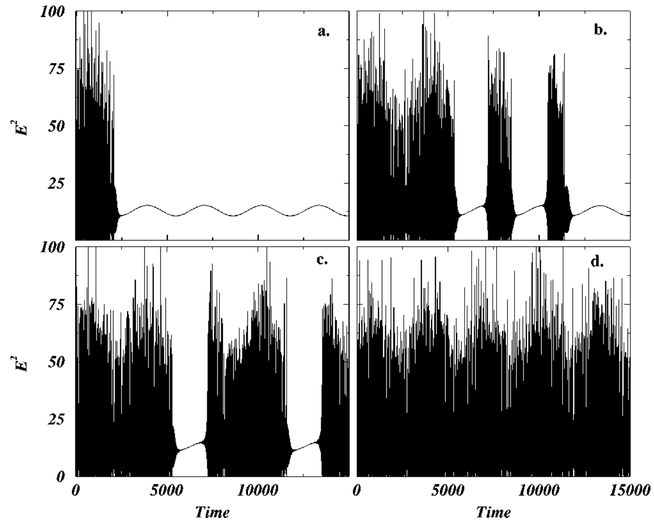

FIG. 8. Numerical results showing the route to chaos as the pump power modulation amplitude is changed. $\sigma = 2.0$, $b = 0.25$, and $r = 14$. (a) has the peak to peak pump power modulation amplitude at 4.6, (b) at 4.2, (c) at 3.8, and (d) at 3.0. The time unit is $\gamma_\perp^{-1}$.

otic fast oscillation described above in the laser between chaos and steady state. This seems to be similar to what is observed in the laser when it is taken through the bifurcations to chaos in a more conventional way [12].

It could be thought that a system one of whose parameters is changing only very slowly compared with its intrinsic dynamics should be in a quasi steady state. However, this is clearly not the case here because the dynamics have obviously been changed radically. As a next level of approximation one might consider modulation with a constant rate of change. Indeed as can be seen in Figs. 9 and 10 a pump power with a constant rate of change does delay the onset of the bifurcation when the pump power is increasing in time. Figure 9 clearly shows the transition to chaos in the laser significantly delayed. In this experiment, the chaos threshold

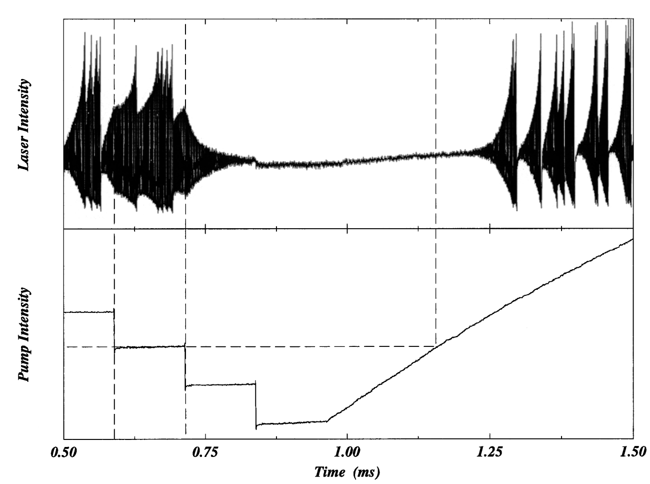

FIG. 9. Time evolution of the output intensity of the laser and the corresponding pump power intensity. The delayed onset of the bifurcation to chaos is clearly present as it is above the constant pump power which is already chaotic. The laser was operated at a pressure of 30 μbar. Amplitude units are in volts, which is proportional to intensity.

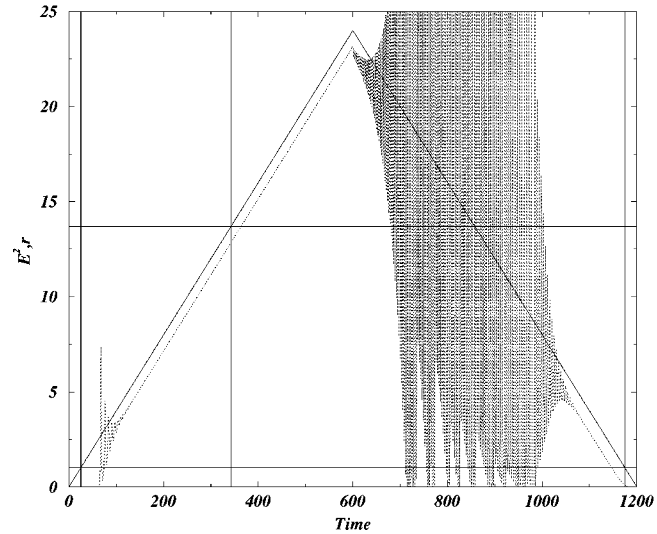

FIG. 10. The time evolution in the nondetuned Lorenz-Haken equations of the output intensity (dotted line) as the pump power (solid line) is ramped up and down. The horizontal lines indicate the lasing threshold and the chaos threshold for the laser. The delayed bifurcations are clearly visible on the positive slope. The negative slope instantaneously spirals out into chaos, showing no stability whatsoever for higher pump powers. The turn off is hard to judge with the preturbulent region which the laser has to negotiate first. The figure therefore gives no clear estimate of the chaos threshold for the negative slope. The time unit is $\gamma_\perp^{-1}$.

for a steady pump can be seen to lie below the middle one of the three levels, while the onset of chaos during the ramp occurs at a considerably higher level, as indicated by the dotted line. Solutions to the equations nearly always show a delay. In the example shown in Fig. 10 the onset of the bifurcation to chaos is delayed sufficiently to be prevented completely for the parameter range shown. Figure 10 also shows that when the pump power has a negative rate of change the stability disappears above the chaos threshold. This has been found over a wide range of the pump power *r* when the rate of change is negative. It could be that the bifurcation is also delayed with the transition to stability occurring at lower pump power, however, this cannot be discerned from the experimental data at present. What can be concluded from this is that one has to be very careful in using approximations to describe slow dynamic changes in potentially chaotic systems.

IV. CONCLUSIONS

We have shown in this paper that by modulating the pump power of the optically pumped far-infrared $^{15}NH_3$ laser, suppression of Lorenz-like chaos is possible as predicted by Vilaseca *et al.* The system tracks the formerly unstable steady state a significant distance into the chaotic regime. If stability is lost it always happens as the pump parameter is dropping rather than on the rise. This is consistent with the behavior when the pump power is ramped up and down with constant rates of change. In every case, we have confirmed that numerical solutions of the complex Lorenz-Haken equations exhibit behavior similar to that observed.

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